Reliable fuzzy control with domain guaranteed cost for fuzzy systems with actuator failures*

JIA Xinchun[1 **] and ZHENG Nanning[2]

(1. Department of Mathematics, Shanxi University, Taiyuan 030006, China; 2. Institute of Artificial Intelligence and Robotics, Xi'an Jiaotong University, Xi'an 710049, China)

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Abstract The reliable fuzzy control with guaranteed cost for T-S fuzzy systems with actuator failure is proposed in this paper. The cost function is a quadratic function with failure input. When the initial state of such systems is known, a design method of the reliable fuzzy controller with reliable guaranteed cost is presented, and the formula of the guaranteed cost is established. When the initial state of such systems is unknown but belongs to a known bounded closed domain, a notion of the reliable domain guaranteed cost (RDGC) for such systems is proposed. For two classes of initial state domain, polygon domain and ellipsoid domain, some design methods for reliable fuzzy controllers with the RDGC are provided. The efficiency of our design methods is finally verified by numerical design and simulation on the Rossler chaotic system.

Keywords: T-S fuzzy systems, reliable fuzzy control, reliable domain guaranteed cost, actuator failures.

In view of fuzzy set and fuzzy inference, the complex nonlinear systems can be efficiently represented by T-S fuzzy systems[1~3], and the stability analysis and controller design methods for T-S fuzzy systems have been studied extensively[3~8].

In the area of guaranteed cost control, the research of quadratic cost index for uncertain systems is one of the important aspects. Since the cost value for uncertain systems cannot be determined precisely, Chang and Peng[9] proposed the problem of fixed upper bound of quadratic cost for such systems, i.e. the guaranteed cost control problem. Consequently, some relative researches have been reported[10~12, 17, 18]. Jadbabaie et al.[8] studied the design method of guaranteed cost controller for continuous fuzzy systems without uncertainties, where the initial state is supposed to be random variable. However, research on guaranteed cost control for T-S fuzzy systems with actuator failure by fuzzy modeling approach have not been reported.

The design purpose of reliable controller is to guarantee that the resulting closed-loop system is tolerant with actuator failure and can retain some useful systematic properties[13~16]. Recently, Yang et al.[17] proposed a general model for actuator failure and studied the reliable guaranteed cost control problem for uncertain nonlinear systems, but its design method is not easy to realize. Jia et al.[18] discussed the reliable dynamic compensator design method for uncertain time varying systems with sensor failure, which is operable.

The following notations will be used in the paper. For given symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, $\boldsymbol{X}<\boldsymbol{Y}$ (or $\boldsymbol{X}\leqslant\boldsymbol{Y}$) denotes $\boldsymbol{Y}-\boldsymbol{X}>0$ (or $\boldsymbol{Y}-\boldsymbol{X}\geqslant 0$), i.e. symmetric positive definite matrix (or positive semi-definite matrix), $\boldsymbol{X}^{\mathrm{T}}$ means the transpose of matrix $\boldsymbol{X}$, I is a real unitary matrix with appropriate dimension, $\operatorname{diag}\{\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots, \boldsymbol{X}_l\}$ is a diagonal block matrix, and $\mathbb{R}^{m\times n}$ is the set of all real matrixces with $m\times n$ dimensions.

1 Problem formulation

Considering the T-S fuzzy systems with actuator failures,

R^i: If $z_1(t)$ is M_{i1}, and $z_g(t)$ is M_{ig}, then

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{u}^{\mathrm{F}}(t), \quad i=1, 2, \cdots, r, \tag{1}$$

where R^i ($i=1, 2, \cdots, r$) denotes the i-th rule of fuzzy uncertain systems (1); $z_1(t), \cdots, z_g(t)$ are precise variables of such fuzzy rules, which are usually state functions; M_{ij} ($i=1, 2, \cdots, r$; $j=1, 2, \cdots, g$) are fuzzy linguistic sets; $\boldsymbol{x}(t)\in\mathbb{R}^n$ and $\boldsymbol{u}^{\mathrm{F}}(t)\in$

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** To whom correspondence should be addressed. E-mail: xchjia@sxu.edu.cn

$\mathbb{R}^m$ are the state vector and the actual input of the systems (1), respectively; $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$ are real constant matrices with appropriate dimensions ($i=1, 2, \cdots, r$). The initial state for fuzzy systems (1) is denoted by $\boldsymbol{x}(0)$. $\boldsymbol{u}_i^{\mathrm{F}}(t)$ is an actual control input and usually an uncertain function of ideal actuator's output $\boldsymbol{u}_i(t)$ ($i=1, 2, \cdots, r$), which is represented by the following actuator failure model

$$u_i^{\mathrm{F}}(t) = \bar{\alpha}_i u_i(t) + \psi_i(u_i), \quad \psi_i^2(u_i) \leqslant \hat{\alpha}_i^2 u_i^2(t),$$
$$i = 1, 2, \cdots, m, \tag{2}$$

where $\psi_i(u_i)$ is the unknown Lebesgue measurable function about u_i, $\bar{\alpha}_i$ and $\hat{\alpha}_i$ are known positive constant numbers with $\bar{\alpha}_i \geqslant \hat{\alpha}_i \geqslant 0$. Let $\bar{\alpha}=\mathrm{diag}\{\bar{\alpha}_1, \bar{\alpha}_2, \cdots, \bar{\alpha}_m\}$, $\hat{\alpha}=\mathrm{diag}\{\hat{\alpha}_1, \hat{\alpha}_2, \cdots, \hat{\alpha}_m\}$, $\boldsymbol{u}^{\mathrm{F}}(t)=[u_1^{\mathrm{F}}, \cdots, u_m^{\mathrm{F}}]^{\mathrm{T}}$, $\psi(\boldsymbol{u})=[\psi_1(u_1), \cdots, \psi_m(u_m)]^{\mathrm{T}}$, then Eq. (2) can be rewritten as

$$\begin{aligned} &\boldsymbol{u}^{\mathrm{F}}(t) = \bar{\alpha}\boldsymbol{u}(t) + \psi(\boldsymbol{u}(t)), \\ &\psi^{\mathrm{T}}(\boldsymbol{u})\,\psi(\boldsymbol{u}) \leqslant \boldsymbol{u}^{\mathrm{T}}(t)\hat{\alpha}^2\boldsymbol{u}(t), \\ &\bar{\alpha} \geqslant \hat{\alpha} \geqslant 0. \end{aligned} \tag{3}$$

When $\bar{\alpha}_i=1$ and $\hat{\alpha}_i=0$, (3) corresponds to the normal case ($u_i^{\mathrm{F}}(t)=u_i(t)$). When $\bar{\alpha}_i=\hat{\alpha}_i$, it is the complete failure case of the i-th actuator. When $0<\hat{\alpha}_i<\bar{\alpha}_i\leqslant 1$, it is the partial failure case.

Let $\boldsymbol{z}=[z_1, z_2, \cdots, z_g]^{\mathrm{T}}$, then the defuzzied output of the fuzzy system (1) is represented as

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))[&\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i(\bar{\alpha}\boldsymbol{u}(t) \\ &+ \psi(\boldsymbol{u}(t)))], \end{aligned} \tag{4}$$

where $\omega_i(\boldsymbol{z}(t))= \prod_{j=1}^{g} \Gamma_j^i(z_j(t))$, $i=1, 2, \cdots, r$,

$$h_i(\boldsymbol{z}(t)) = \frac{\omega_i(\boldsymbol{z}(t))}{\sum_{j=1}^{r}\omega_j(z(t))}, \quad i = 1, 2, \cdots, r,$$

$\Gamma_j^i(z_j(t))$ is the membership function of $z_j(t)$ in M_i^j ($i=1, 2, \cdots, r$), and $h_i(\boldsymbol{z}(t))$ ($i=1, 2, \cdots, r$) is the normalized membership functions of system (1), which satisfy the following properties:

$$h_i(t) \geqslant 0, \quad i = 1, 2, \cdots, r, \quad \sum_{i=1}^{r} h_i(t) = 1. \tag{5}$$

For simplicity, denote $h_i(\boldsymbol{z}(t))$ by $h_i(t)$ (or h_i) ($i=1, 2, \cdots, r$) in this paper.

For the fuzzy system (1), the fuzzy controller based on the PDC technique is chosen to be

C^i: If $z_1(t)$ is M_{i1} and $\cdots$ and $z_g(t)$ is M_{ig}; then

$$\boldsymbol{u}(t) = \boldsymbol{F}_i\boldsymbol{x}(t), \quad i = 1, 2, \cdots, r, \tag{6}$$

then, the global fuzzy controller is a nonlinear controller, i.e.

$$\boldsymbol{u}(t) = \sum_{i=1}^{r} h_i(t)\boldsymbol{F}_i\boldsymbol{x}(t). \tag{7}$$

The resulting closed-loop system is

$$\begin{aligned} \dot{\boldsymbol{x}}(t) = &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j(\boldsymbol{A}_i + \boldsymbol{B}_i\bar{\alpha}\boldsymbol{F}_j)\boldsymbol{x}(t) \\ &+ \sum_{i=1}^{r} h_i\boldsymbol{B}_i\,\psi\left(\sum_{j=1}^{r} h_j\boldsymbol{F}_j\boldsymbol{x}(t)\right). \end{aligned} \tag{8}$$

The cost function for fuzzy systems (1) is chosen as a quadratic cost function with failure input, i.e.

$$J = \int_0^{+\infty} [\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{R}_1\boldsymbol{x}(t) + (\boldsymbol{u}^{\mathrm{F}}(t))^{\mathrm{T}}\boldsymbol{R}_2(\boldsymbol{u}^{\mathrm{F}}(t))]\,\mathrm{d}t, \tag{9}$$

where $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are given symmetric positive definite matrices, and $\boldsymbol{R}_2$ is a diagonal matrix.

The problems under consideration in the paper are formulated as follows:

(I) Given the known initial state for fuzzy systems (1), the design purpose of fuzzy control law (6) is to guarantee that the closed-loop system (8) is robustly quadratic stable and the cost function (9) of system (8) satisfies $J\leqslant J_0$ for any admissible actuator failure (2), where J_0 is a positive constant number. If such conditions hold, then the controller (6) is referred to as a reliable fuzzy controller with a reliable guaranteed cost J_0 for fuzzy systems (1).

(II) When the initial state of fuzzy systems (1) is unknown but belongs to a bounded closed domain Ω (called the initial state domain), the design purpose of the fuzzy control law (6) is to guarantee that the closed-loop system (8) is robustly quadratic stable for any admissible actuator failure (2), and the cost function (9) of system (8) satisfies $J\leqslant J(\Omega)$ for any admissible actuator failure (2) and any admissible initial state $x(0)\in\Omega$, where $J(\Omega)$ is a positive constant number. If such conditions are satisfied, the controller (6) is called a reliable fuzzy controller with a reliable domain guaranteed cost (abbreviated to RDGC) $J(\Omega)$ for fuzzy systems (1) with the initial domain Ω.

Lemma 1[6]. Given some real vectors $\boldsymbol{y}_i\in\mathbb{R}^m$ ($i=1, 2, \cdots, s$), and a symmetric positive definite matrix $\boldsymbol{M} \in \mathbb{R}^{m \times m}$. If $\sum_{i=1}^{s} p_i = 1$ and $0 \leqslant p_i \leqslant 1$, then $\left(\sum_{i=1}^{s} p_i \boldsymbol{y}_i\right)^{\mathrm{T}} \boldsymbol{M} \left(\sum_{i=1}^{s} p_i \boldsymbol{y}_i\right) \leqslant \sum_{i=1}^{s} p_i \boldsymbol{y}_i^{\mathrm{T}} \boldsymbol{M} \boldsymbol{y}_i$.

2 Design method of reliable fuzzy controller and reliable guaranteed cost

In this section, we assume that the initial state of fuzzy system (1) is known. A design method for reliable fuzzy controller, and a formula of reliable guaranteed cost are presented. Meanwhile, we will establish an optimal technique to decrease the conservatism of reliable guaranteed cost.

Theorem 1. Given fuzzy systems (1), actuator failure model (2) and quadratic cost function (9), if there exists a feasible solution to matrix inequalities (10) ~ (12): a symmetric positive definite matrix $\boldsymbol{Z} \in \mathbb{R}^{n \times n}$, matrices $\boldsymbol{Q}_i \in \mathbb{R}^{m \times n}$ $(i = 1, 2, \cdots, r)$, $\boldsymbol{Y}_{ij} \in \mathbb{R}^{n \times n}$ $(i, j = 1, 2, \cdots, r)$, positive real numbers $\varepsilon_i (i = 1, 2, \cdots, r)$ and $\bar{\varepsilon}$, where $\boldsymbol{Y}_{ji} = \boldsymbol{Y}_{ij}^{\mathrm{T}} (i \neq j)$, then the controller (6) with gain matrices $\boldsymbol{F}_i = \boldsymbol{Q}_i \boldsymbol{Z}^{-1} (i = 1, 2, \cdots, r)$ is a reliable fuzzy controller with a guaranteed cost $J(\boldsymbol{x}(0)) = \boldsymbol{x}^{\mathrm{T}}(0) \boldsymbol{Z}^{-1} \boldsymbol{x}(0)$ for fuzzy systems (1):

$$
\begin{aligned}
&[(\boldsymbol{Z}\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{Q}_i^{\mathrm{T}} \bar{\alpha} \boldsymbol{B}_i^{\mathrm{T}}) + (\boldsymbol{A}_i \boldsymbol{Z} + \boldsymbol{B}_i \bar{\alpha} \boldsymbol{Q}_i) \\
&\quad + \varepsilon_i \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} + \varepsilon_i^{-1} \boldsymbol{Q}_i^{\mathrm{T}} \hat{\alpha}^2 \boldsymbol{Q}_i] \\
&\quad + [\boldsymbol{Z}\boldsymbol{R}_1 \boldsymbol{Z} + \boldsymbol{Q}_i^{\mathrm{T}} (\bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} + \hat{\alpha} \boldsymbol{R}_2 \hat{\alpha} \\
&\quad + \bar{\varepsilon} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} + \bar{\varepsilon}^{-1} \hat{\alpha} \boldsymbol{R}_2 \hat{\alpha}) \boldsymbol{Q}_i] \leqslant \boldsymbol{Y}_{ii}, \\
&\qquad i = 1, 2, \cdots, r,
\end{aligned} \tag{10}
$$

$$
\begin{aligned}
&[(\boldsymbol{Z}\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{Q}_j \bar{\alpha} \boldsymbol{B}_j^{\mathrm{T}} + \boldsymbol{Z}\boldsymbol{A}_j^{\mathrm{T}} + \boldsymbol{Q}_i \bar{\alpha} \boldsymbol{B}_j^{\mathrm{T}}) \\
&\quad + (\boldsymbol{A}_i \boldsymbol{Z} + \boldsymbol{B}_i \bar{\alpha} \boldsymbol{Q}_j + \boldsymbol{A}_j \boldsymbol{Z} + \boldsymbol{B}_j \bar{\alpha} \boldsymbol{Q}_i) + \varepsilon_i \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} \\
&\quad + \varepsilon_j \boldsymbol{B}_j \boldsymbol{B}_j^{\mathrm{T}} + \varepsilon_i^{-1} \boldsymbol{Q}_j^{\mathrm{T}} \hat{\alpha}^2 \boldsymbol{Q}_j + \varepsilon_j^{-1} \boldsymbol{Q}_i^{\mathrm{T}} \hat{\alpha}^2 \boldsymbol{Q}_i] \\
&\quad + [2\boldsymbol{Z}\boldsymbol{R}_1 \boldsymbol{Z} + \boldsymbol{Q}_i^{\mathrm{T}} (\bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} + \bar{\varepsilon} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} \\
&\quad + \hat{\alpha} \boldsymbol{R}_2 \hat{\alpha} + \bar{\varepsilon}^{-1} \hat{\alpha} \boldsymbol{R}_2 \hat{\alpha}) \boldsymbol{Q}_i + \boldsymbol{Q}_j^{\mathrm{T}} (\bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} \\
&\quad + \bar{\varepsilon} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} + \hat{\alpha} \boldsymbol{R}_2 \hat{\alpha} + \bar{\varepsilon}^{-1} \hat{\alpha} \boldsymbol{R}_2 \hat{\alpha}) \boldsymbol{Q}_j] \\
&\leqslant \boldsymbol{Y}_{ij} + \boldsymbol{Y}_{ij}^{\mathrm{T}}, \quad i < j,
\end{aligned} \tag{11}
$$

$$
\boldsymbol{Y} = [\boldsymbol{Y}_{ij}]_{r \times r} < 0. \tag{12}
$$

Proof. Let $\boldsymbol{P} = \boldsymbol{Z}^{-1}$, $\boldsymbol{F}_i = \boldsymbol{Q}_i \boldsymbol{Z}^{-1} (i = 1, 2, \cdots, r)$. Choosing $V(t) = \boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{P} \boldsymbol{x}(t)$ as a Lyapunov function candidate for system (8), the time derivative of $V(t)$ along the solution trajectories is given by

$$
\begin{aligned}
\dot{V}(t) &= \dot{\boldsymbol{x}}^{\mathrm{T}}(t) \boldsymbol{P} \boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{P} \dot{\boldsymbol{x}}(t) \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \boldsymbol{x}^{\mathrm{T}}(t) [(\boldsymbol{A}_i + \boldsymbol{B}_i \bar{\alpha} \boldsymbol{F}_j)^{\mathrm{T}} \boldsymbol{P} \\
&\quad + \boldsymbol{P}(\boldsymbol{A}_i + \boldsymbol{B}_i \bar{\alpha} \boldsymbol{F}_j)] \boldsymbol{x}(t) \\
&\quad + \sum_{i=1}^{r} h_i 2\boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{P} \boldsymbol{B}_i \psi\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right).
\end{aligned}
$$

In view of Lemma 1, we have

$$
\begin{aligned}
&\sum_{i=1}^{r} h_i 2\boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{P} \boldsymbol{B}_i \psi\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right) \\
&\leqslant \sum_{i=1}^{r} h_i \Big[\varepsilon_i \boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{P} \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{x}(t) \\
&\quad + \varepsilon_i^{-1} \psi^{\mathrm{T}}\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right) \psi\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right)\Big] \\
&\leqslant \sum_{i=1}^{r} h_i \Big[\boldsymbol{x}^{\mathrm{T}}(t) \varepsilon_i \boldsymbol{P} \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P} \boldsymbol{x}(t) \\
&\quad + \varepsilon_i^{-1} \left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right)^{\mathrm{T}} \hat{\alpha}^2 \left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right)\Big] \\
&\leqslant \sum_{i=1}^{r} h_i \boldsymbol{x}^{\mathrm{T}}(t) \Big[\varepsilon_i \boldsymbol{P} \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P} \\
&\quad + \varepsilon_i^{-1} \sum_{j=1}^{r} h_j \boldsymbol{F}_j^{\mathrm{T}} \hat{\alpha}^2 \boldsymbol{F}_j\Big] \boldsymbol{x}(t) \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \boldsymbol{x}^{\mathrm{T}}(t) [\varepsilon_i \boldsymbol{P} \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P} \\
&\quad + \varepsilon_i^{-1} \boldsymbol{F}_j^{\mathrm{T}} \hat{\alpha}^2 \boldsymbol{F}_j] \boldsymbol{x}(t).
\end{aligned}
$$

Hence, we obtain the following inequality

$$
\begin{aligned}
\dot{V}(t) \leqslant &\sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \boldsymbol{x}^{\mathrm{T}}(t) [(\boldsymbol{A}_i + \boldsymbol{B}_i \bar{\alpha} \boldsymbol{F}_j)^{\mathrm{T}} \boldsymbol{P} \\
&\quad + \boldsymbol{P}(\boldsymbol{A}_i + \boldsymbol{B}_i \bar{\alpha} \boldsymbol{F}_j) + \varepsilon_i \boldsymbol{P} \boldsymbol{B}_i \boldsymbol{B}_i^{\mathrm{T}} \boldsymbol{P} \\
&\quad + \varepsilon_i^{-1} \boldsymbol{F}_j^{\mathrm{T}} \hat{\alpha}^2 \boldsymbol{F}_j] \boldsymbol{x}(t).
\end{aligned} \tag{13}
$$

On the other hand, the integral function in cost function (9) can be rewritten as

$$
\begin{aligned}
&\boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{R}_1 \boldsymbol{x}(t) + (\boldsymbol{u}^{\mathrm{F}}(t))^{\mathrm{T}} \boldsymbol{R}_2 (\boldsymbol{u}^{\mathrm{F}}(t)) \\
&= [\boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{R}_1 \boldsymbol{x}(t) \\
&\quad + (\bar{\alpha} \boldsymbol{u}(t) + \psi(\boldsymbol{u}))^{\mathrm{T}} \boldsymbol{R}_2 (\bar{\alpha} \boldsymbol{u}(t) + \psi(\boldsymbol{u}))] \\
&= \boldsymbol{x}^{\mathrm{T}}(t) \boldsymbol{R}_1 \boldsymbol{x}(t) + \boldsymbol{x}^{\mathrm{T}}(t) \left(\sum_{i=1}^{r} h_i \boldsymbol{F}_i\right)^{\mathrm{T}} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} \\
&\quad \circ \left(\sum_{i=1}^{r} h_i \boldsymbol{F}_i\right) \boldsymbol{x}(t) \\
&\quad + 2\left(\sum_{i=1}^{r} h_i \boldsymbol{F}_i \boldsymbol{x}(t)\right)^{\mathrm{T}} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \psi\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right) \\
&\quad + \psi^{\mathrm{T}}\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right) \boldsymbol{R}_2 \psi\left(\sum_{j=1}^{r} h_j \boldsymbol{F}_j \boldsymbol{x}(t)\right).
\end{aligned} \tag{14}
$$

By (3) and Lemma 1, the second, third and fourth terms in the right of Eq. (14) can be dealt with as

$$
\begin{aligned}
&\left(\sum_{i=1}^{r} h_i \boldsymbol{F}_i\right)^{\mathrm{T}} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} \left(\sum_{i=1}^{r} h_i \boldsymbol{F}_i\right) \\
&\leqslant \sum_{i=1}^{r} h_i \boldsymbol{F}_i^{\mathrm{T}} \bar{\alpha}^{\mathrm{T}} \boldsymbol{R}_2 \bar{\alpha} \boldsymbol{F}_i,
\end{aligned}
$$

$$\psi^{T}\left(\sum_{j=1}^{r}h_jF_jx(t)\right)R_2\,\psi\left(\sum_{j=1}^{r}h_jF_jx(t)\right)$$
$$\leqslant\left(\sum_{j=1}^{r}h_jF_jx(t)\right)^{T}\hat{\alpha}R_2\hat{\alpha}\left(\sum_{j=1}^{r}h_jF_jx(t)\right)$$
$$\leqslant\sum_{i=1}^{r}h_ix^{T}(t)F_i^{T}\hat{\alpha}R_2\hat{\alpha}F_ix(t),$$

$$2\left(\sum_{i=1}^{r}h_iF_ix(t)\right)^{T}\bar{\alpha}^{T}R_2\,\psi\left(\sum_{j=1}^{r}h_jF_jx(t)\right)$$
$$=2\sum_{i=1}^{r}h_ix^{T}(t)F_i^{T}\bar{\alpha}^{T}R_2\,\psi\left(\sum_{j=1}^{r}h_jF_jx(t)\right)$$
$$\leqslant\sum_{i=1}^{r}h_i\left\{\bar{\varepsilon}x(t)^{T}F_i^{T}\bar{\alpha}^{T}R_2\bar{\alpha}F_ix(t)+\bar{\varepsilon}^{-1}\psi^{T}\left(\sum_{j=1}^{r}h_jF_jx(t)\right)R_2\,\psi\left(\sum_{j=1}^{r}h_jF_jx(t)\right)\right\}$$
$$\leqslant\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jx^{T}(t)[\bar{\varepsilon}F_i^{T}\bar{\alpha}^{T}R_2\bar{\alpha}F_i+\bar{\varepsilon}^{-1}F_j^{T}\hat{\alpha}R_2\hat{\alpha}F_j]\,x(t),$$

and then we obtain

$$x^{T}(t)R_1x(t)+(u^{F}(t))^{T}R_2(u^{F}(t))$$
$$\leqslant x^{T}(t)\left[R_1+\sum_{i=1}^{r}h_iF_i^{T}\bar{\alpha}^{T}R_2\bar{\alpha}F_i\right]x(t)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jx^{T}(t)[\bar{\varepsilon}F_i^{T}\bar{\alpha}^{T}R_2\bar{\alpha}F_i+\bar{\varepsilon}^{-1}F_j^{T}\hat{\alpha}R_2\hat{\alpha}F_j]\,x(t)$$
$$+\sum_{i=1}^{r}h_ix^{T}(t)F_i^{T}\hat{\alpha}R_2\hat{\alpha}F_ix(t)$$
$$=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_jx^{T}(t)[R_1+F_i^{T}(\bar{\alpha}^{T}R_2\bar{\alpha}+\bar{\varepsilon}\,\bar{\alpha}^{T}R_2\bar{\alpha}+\hat{\alpha}R_2\hat{\alpha})F_i+\bar{\varepsilon}^{-1}F_j^{T}\hat{\alpha}R_2\hat{\alpha}F_j]\,x(t).$$

By the above inequalities, we conclude that

$$\mathring{V}(t)+\{x^{T}(t)R_1x(t)+(u^{F}(t))^{T}R_2(u^{F}(t))\}$$
$$\leqslant\sum_{i=1}^{r}h_i^{2}x^{T}(t)\{[(A_i+B_i\bar{\alpha}F_i)^{T}P+P(A_i+B_i\bar{\alpha}F_i)+\varepsilon_iPB_iB_i^{T}P+\varepsilon_i^{-1}F_i^{T}\hat{\alpha}^{2}F_i]$$
$$+[R_1+F_i^{T}(\bar{\alpha}^{T}R_2\bar{\alpha}+\bar{\varepsilon}\,\bar{\alpha}^{T}R_2\bar{\alpha}+\hat{\alpha}R_2\hat{\alpha})F_i+\bar{\varepsilon}^{-1}F_i^{T}\hat{\alpha}R_2\hat{\alpha}F_i]\}\,x(t)$$
$$+\sum_{i=1}^{r}\sum_{i<j}^{r}h_ih_jx^{T}(t)\{[(A_i+B_i\bar{\alpha}F_j+A_j+B_j\bar{\alpha}F_i)^{T}P+P(A_i+B_i\bar{\alpha}F_j+A_j+B_j\bar{\alpha}F_i)$$
$$+\varepsilon_iPB_iB_i^{T}P+\varepsilon_jPB_jB_j^{T}P+\varepsilon_i^{-1}F_j^{T}\hat{\alpha}^{2}F_j+\varepsilon_j^{-1}F_i^{T}\hat{\alpha}^{2}F_i]$$
$$+[2R_1+F_i^{T}(\bar{\alpha}^{T}R_2\bar{\alpha}+\bar{\varepsilon}\,\bar{\alpha}^{T}R_2\bar{\alpha}+\hat{\alpha}R_2\hat{\alpha})F_i+F_j^{T}(\bar{\alpha}^{T}R_2\bar{\alpha}+\bar{\varepsilon}\,\bar{\alpha}^{T}R_2\bar{\alpha}+\hat{\alpha}R_2\hat{\alpha})F_j$$
$$+\bar{\varepsilon}^{-1}F_j^{T}\hat{\alpha}R_2\hat{\alpha}F_j+\bar{\varepsilon}^{-1}F_i^{T}\hat{\alpha}R_2\hat{\alpha}F_i]\}\,x(t).\quad(15)$$

Premultiply and postmultiply (10) and (11) by P, we can derive from (12) and (15) that

$$\mathring{V}(t)+\{x^{T}(t)R_1x(t)+(u^{F}(t))^{T}R_2(u^{F}(t))\}\leqslant\begin{bmatrix}h_1x\\ \vdots\\ h_nx\end{bmatrix}\begin{bmatrix}PY_{11}P&\cdots&PY_{1n}P\\ \vdots&\ddots&\vdots\\ PY_{n1}P&\cdots&PY_{nn}P\end{bmatrix}\begin{bmatrix}h_1x\\ \vdots\\ h_nx\end{bmatrix}<0\quad(16)$$

holds for any nonzero vector $x(t)\in\mathbb{R}^{n}$ and any admissible actuator failure (2). Therefore, the closed-loop fuzzy system (8) is robustly quadratic stable.

From (16), we have $\{x^{T}(t)R_1x(t)+(u^{F}(t))^{T}R_2(u^{F}(t))\}\leqslant-\mathring{V}(t)$. Integrating the above inequality from $0+\infty$ in both sides simultaneously, and noting that system (8) is robustly quadratic stable, we can obtain $J\leqslant V(0)=x^{T}(0)Z^{-1}x(0)=J_0$. This completes the proof of Theorem 1.

When fuzzy system (1) is failure-free, i.e. $\bar{\alpha}=1$, $\hat{\alpha}=0$, let $\bar{\varepsilon}_i\to0^{+}$ in (10) and (11), we can obtain the result which accords with the case that fuzzy system (1) is not with actuator failures.

In Theorem 1, the matrix inequalities (10) and (11) with respect to variables Z, Q_i ($i=1,2,\cdots,r$), ε_i ($i=1,2,\cdots,r$) and $\bar{\varepsilon}$ are not LMIs, which cannot be solved directly by LMI approach. Let $R(\bar{\varepsilon})=\bar{\alpha}^{T}R_2\bar{\alpha}+\bar{\varepsilon}\,\bar{\alpha}^{T}R_2\bar{\alpha}+\hat{\alpha}R_2\hat{\alpha}+\bar{\varepsilon}^{-1}\hat{\alpha}R_2\hat{\alpha}$. In order to solve the matrix inequalities (10)~(12) efficiently, we introduce a minimal procedure for $R(\bar{\varepsilon})$ with variable $\bar{\varepsilon}$: $\min\{\mathrm{trace}(R(\bar{\varepsilon})):\bar{\varepsilon}>0\}$, and obtain the constant number $\bar{\varepsilon}$, then we can get $R(\bar{\varepsilon})$ by $\bar{\varepsilon}$, and briefly denote it by R. Consequently, the matrix inequalities (10) and (11) can be transformed into some LMIs.

By Schur complementary principle, inequalities (10) are equivalent to the following LMIs,

$$\begin{bmatrix}\Xi_{ii}&Q_i^{T}\hat{\alpha}&Q_i^{T}R&ZR_i\\ \hat{\alpha}Q_i&-\varepsilon_iI&0&0\\ RQ_i&0&-R&0\\ R_1Z&0&0&-R_1\end{bmatrix}<0,$$
$$i=1,2,\cdots,r,\quad(17)$$

where $\Xi_{ii}=[(ZA_i^{T}+Q_i^{T}\bar{\alpha}B_i^{T})+(A_iZ+B_i\bar{\alpha}Q_i)+\varepsilon_iB_iB_i^{T}-Y_{ii}]$, $i=1,2,\cdots,r$.

In the same way, inequalities (11) are equivalent to the following LMIs,

$$\begin{bmatrix} \Sigma_{ij} & \boldsymbol{Q}_j^{\mathrm{T}}\hat{\alpha} & \boldsymbol{Q}_i^{\mathrm{T}}\hat{\alpha} & \boldsymbol{Z}\boldsymbol{R}_1 & \boldsymbol{Q}_i^{\mathrm{T}}\boldsymbol{R} & \boldsymbol{Q}_j^{\mathrm{T}}\boldsymbol{R} \\ \hat{\alpha}\boldsymbol{Q}_j & -\varepsilon_i I & 0 & 0 & 0 & 0 \\ \hat{\alpha}\boldsymbol{Q}_i & 0 & -\varepsilon_j I & 0 & 0 & 0 \\ \boldsymbol{R}_1\boldsymbol{Z} & 0 & 0 & -0.5\boldsymbol{R}_1 & 0 & 0 \\ \boldsymbol{R}\boldsymbol{Q}_i & 0 & 0 & 0 & -\boldsymbol{R} & 0 \\ \boldsymbol{R}\boldsymbol{Q}_j & 0 & 0 & 0 & 0 & -\boldsymbol{R} \end{bmatrix} < 0, \quad i < j, \tag{18}$$

where

$$\begin{aligned} \Sigma_{ij} = & \boldsymbol{Z}\boldsymbol{A}_i^{\mathrm{T}} + \boldsymbol{Q}_j\bar{\alpha}\boldsymbol{B}_i^{\mathrm{T}} + \boldsymbol{Z}\boldsymbol{A}_j^{\mathrm{T}} + \boldsymbol{Q}_i\bar{\alpha}\boldsymbol{B}_j^{\mathrm{T}} \\ & + \boldsymbol{A}_i\boldsymbol{Z} + \boldsymbol{B}_i\bar{\alpha}\boldsymbol{Q}_j + \boldsymbol{A}_j\boldsymbol{Z} + \boldsymbol{B}_j\bar{\alpha}\boldsymbol{Q}_i \\ & + \varepsilon_i\boldsymbol{B}_i\boldsymbol{B}_i^{\mathrm{T}} + \varepsilon_j\boldsymbol{B}_j\boldsymbol{B}_j^{\mathrm{T}} - \boldsymbol{Y}_{ij} - \boldsymbol{Y}_{ij}^{\mathrm{T}}, \quad i < j. \end{aligned}$$

In sum, the design procedure of the reliable fuzzy controller (6) with a reliable guaranteed cost can be completed in three steps. Firstly, compute a constant number $\bar{\varepsilon}$ and a symmetric matrix $\boldsymbol{R}$ by the minimum technique $\min\{\mathrm{trace}(\boldsymbol{R}(\bar{\varepsilon})): \bar{\varepsilon} > 0\}$. Secondly, solve the LMIs (12) (18) (19) and obtain a feasible solution $\boldsymbol{Z} \in \mathbb{R}^{n\times n}$, $\boldsymbol{Q}_i \in \mathbb{R}^{m\times n}$ ($i = 1, 2, \cdots, r$), etc. Finally, obtain a reliable fuzzy controller (6) with gain matrices $\boldsymbol{F}_i = \boldsymbol{Q}_i\boldsymbol{Z}^{-1}$ ($i = 1, 2, \cdots, r$), and a reliable guaranteed cost $J(x(0))$.

If there exists a feasible solution of the LMIs (12) (17) (18), then there exist many feasible solutions of them. Hence, one can decrease the conservatism of such reliable guaranteed cost by choosing an appropriate feasible solution of the LMIs (12) (17) (18). This idea can be realized by the minimum procedure of positive constant number ρ subject to $\boldsymbol{x}^{\mathrm{T}}(0)\boldsymbol{Z}^{-1}\boldsymbol{x}(0) < \rho$, which can be rewritten as

$$\begin{bmatrix} -\rho & \boldsymbol{x}^{\mathrm{T}}(0) \\ \boldsymbol{x}(0) & -\boldsymbol{Z} \end{bmatrix} < 0. \tag{19}$$

Consequently, we propose the following linear convex optimal method

$$\begin{cases} \min\{J(\rho) = \rho: \boldsymbol{Z} > 0, \rho > 0\}, \\ \text{Subject to: LMIs (12), (17), (18), and (19),} \end{cases} \tag{20}$$

and denote the reliable guaranteed cost corresponding to the optimal solution to (20) by $J^*(\boldsymbol{x}(0))$.

3 Design of reliable fuzzy controller with domain guaranteed cost

In general, the initial state for fuzzy systems (1) is usually unknown. However, in view of practical situation of systems (1), the domain that the initial state belongs to is always determined. Hence, we assume that the initial state $x(0)$ belongs to a bounded closed domain Ω in this section, which is called the initial state domain. Define $J(\Omega) = \max\{\boldsymbol{x}^{\mathrm{T}}(0)\boldsymbol{Z}^{-1}\boldsymbol{x}(0): \boldsymbol{x}(0) \in \Omega\}$ as a reliable domain guaranteed cost (RDGC) with respect to ($w.r.t.$) Ω for fuzzy systems (1), where $\boldsymbol{Z} \in \mathbb{R}^{n\times n}$ is a feasible solution for LMIs (12) (17) (18).

3.1 Design of reliable fuzzy controller and reliable domain guaranteed cost $w.r.t.$ Ω_1

Considering the case that the initial state domain of fuzzy systems is a known polygon domain, i.e.

$$\Omega_1 = \mathrm{co}\{\boldsymbol{x}_{01}, \boldsymbol{x}_{02}, \cdots, \boldsymbol{x}_{0s}\}, \tag{21}$$

it is easy to see that

$$\Omega_1 = \left\{ \sum_{i=1}^{s} p_i\boldsymbol{x}_{0i}: \sum_{i=1}^{s} p_i = 1,\ p_i \in [0, 1],\ i = 1, 2, \cdots, s \right\}.$$

Let $\hat{\Omega}_1 = \left\{ p := (p_1, p_2, \cdots, p_s): \sum_{i=1}^{s} p_i = 1, p_i \in [0, 1],\ i = 1, 2, \cdots, s \right\}$, then we have

$$\max\{J_0 = \boldsymbol{x}^{\mathrm{T}}(0)\boldsymbol{P}\boldsymbol{x}(0): \boldsymbol{x}(0) \in \Omega_1\} = \max\left\{ \left(\sum_{i=1}^{s} p_i\boldsymbol{x}_{0i}\right)^{\mathrm{T}}\boldsymbol{P}\left(\sum_{i=1}^{s} p_i\boldsymbol{x}_{0i}\right) : p \in \hat{\Omega}_1 \right\}.$$

By Lemma 1, we can derive that

$$\begin{aligned} & \max\left\{ \left(\sum_{i=1}^{s} p_i\boldsymbol{x}_{0i}\right)^{\mathrm{T}}\boldsymbol{P}\left(\sum_{i=1}^{s} p_i\boldsymbol{x}_{0i}\right) : p \in \hat{\Omega}_1 \right\} \\ & \leqslant \max\left\{ \sum_{i=1}^{s} p_i\boldsymbol{x}_{0i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_{0i}: p \in \hat{\Omega}_1 \right\} \\ & \leqslant \max_{1\leqslant i\leqslant s}\{\boldsymbol{x}_{0i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_{0i}\}. \end{aligned}$$

Since $\boldsymbol{x}_{0i} \in \Omega_1$ ($i = 1, 2, \cdots, s$) hold, then $J(\Omega_1) = \max_{1\leqslant i\leqslant s}\{\boldsymbol{x}_{0i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_{0i}\}$.

Theorem 2. Given fuzzy systems (1) with initial state domain Ω_1 and quadratic cost function (9). If there exists a feasible solution to matrix inequalities (12) (17) (18) (a symmetric positive definite matrix $\boldsymbol{Z} \in \mathbb{R}^{n\times n}$, matrices $\boldsymbol{Q}_i \in \mathbb{R}^{m\times n}$ ($i = 1, 2, \cdots, r$), $\boldsymbol{Y}_{ij} \in \mathbb{R}^{n\times n}$ ($i, j = 1, 2, \cdots, r$), and positive real numbers ε_i ($i = 1, 2, \cdots, r$), where $\boldsymbol{Y}_{ji} = \boldsymbol{Y}_{ij}^{\mathrm{T}}$ ($i \neq j$)), then the controller (6) with gain matrices $\boldsymbol{F}_i = \boldsymbol{Q}_i\boldsymbol{P}$ ($\boldsymbol{P} = \boldsymbol{Z}^{-1}$) ($i = 1, 2, \cdots, r$) is a reliable fuzzy controller with domain guaranteed cost $J(\Omega_1)$ for fuzzy systems (1), and a RDGC is $J(\Omega_1) = \max_{1\leqslant i\leqslant s}\{\boldsymbol{x}_{0i}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}_{0i}\}$.

Remark 1. The conservatism of RDGC for fuzzy systems (1) can be deceased by choosing an appropriate feasible solution to LMIs (12) (17) (18). The minimum problem of $J(\Omega_1) = \max_{1 \leqslant i \leqslant s}\{x_{0i}^{\mathrm{T}} P x_{0i}\}$ can be solved by the minimization of the positive number γ subject to $x_{0i}^{\mathrm{T}} P x_{0i} \leqslant \gamma$ ($i = 1, 2, \cdots, s$). $x_{0i}^{\mathrm{T}} P x_{0i} \leqslant \gamma$ ($i = 1, 2, \cdots, s$) is equivalent to

$$\begin{bmatrix} -\gamma & x_{0i}^{\mathrm{T}} \\ x_{0i} & -Z \end{bmatrix} \leqslant 0, \quad (i = 1, 2, \cdots, s). \tag{22}$$

By the above analysis, we propose an optimal problem

$$\begin{cases} \min\limits_{Z > 0,\, \gamma > 0}: J(\gamma) = \gamma, \\ \text{Subject to: LMIs (12), (17), (18), (22).} \end{cases} \tag{23}$$

Solving the optimal problem (23), we can obtain the RDGC with least conservative for fuzzy systems (1), and denote it by $J^*(\Omega_1)$.

3.2 Design of reliable fuzzy controller with reliable domain guaranteed cost $w.r.t.$ Ω_2

Consider that the initial state domain of fuzzy systems (1) is a known ellipsoid domain, i.e.

$$\Omega_2 = \{x : x^{\mathrm{T}} R x \leqslant 1, x \in \mathbb{R}^n, R > 0\}, \tag{24}$$

let $\Omega_\rho = \{x : x^{\mathrm{T}} P x \leqslant \rho, \rho > 0, P = Z^{-1}\}$, it is easy to derive

$$\begin{aligned} J(\Omega_2) &= \max\{J_0 = x^{\mathrm{T}} P x : x \in \Omega_2\} \\ &= \min\{\rho : \rho > 0, \Omega_2 \subset \Omega_\rho\} \\ &= \min\{\rho : \rho > 0, P \leqslant \rho R\}. \end{aligned} \tag{25}$$

Theorem 3. Given fuzzy systems (1) with initial state domain Ω_2 and quadratic cost function (9). If there exists a feasible solution to matrix inequalities (12) (17) (18) (a symmetric positive definite matrix $Z \in \mathbb{R}^{n \times n}$, matrices $Q_i \in \mathbb{R}^{m \times n}$ ($i = 1, 2, \cdots, r$), $Y_{ij} \in \mathbb{R}^{n \times n}$ ($i, j = 1, 2, \cdots, r$), and positive real numbers ε_i ($i = 1, 2, \cdots, r$), where $Y_{ji} = Y_{ij}^{\mathrm{T}}$ ($i \neq j$)), then the controller (6) with gain matrices $F_i = Q_i P$ ($P = Z^{-1}$) ($i = 1, 2, \cdots, r$) is a reliable fuzzy controller with RDGC $J(\Omega_2)$ for fuzzy systems (1), and a RDGC is $J(\Omega_2) = \min\{\rho : \rho > 0, P \leqslant \rho R\}$.

The inequality $P \leqslant \rho R$ is equivalent to (26). We propose a linear convex optimal problem (27), which can efficiently decrease the conservatism of $J(\Omega_2)$. By solving the optimal problem, we can obtain the RDGC with least conservatism, and denote it by $J^*(\Omega_2)$.

$$\begin{bmatrix} -Z & I \\ I & -\rho R \end{bmatrix} \leqslant 0. \tag{26}$$

$$\begin{cases} \min\limits_{Z > 0,\, \rho > 0}: J(\rho) = \rho, \\ \text{Subject to: LMIs (12), (17), (18), (26).} \end{cases} \tag{27}$$

4 Simulations

Considering the Rossler system with actuator failure[4]

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t), \\ \dot{x}_2(t) = x_1(t) + a x_2(t), \\ \dot{x}_3(t) = b x_1(t) - (c - x_1(t)) x_3(t) + u^{\mathrm{F}}(t), \end{cases} \tag{28}$$

where $a = 0.34$, $b = 0.4$, and $c = 4.5$, $u^{\mathrm{F}}(t)$ is the output of failure actuator, and also the actual input for the systems (28), the parameters in the actuator failure model (2) are $\bar{\alpha} = 0.9$ and $\hat{\alpha} = 0.1$, and the weighted matrices in the cost function (9) are $R_1 = \mathrm{diag}\{0.5, 1, 0.8\}$, $R_2 = 1$. When $u^{\mathrm{F}}(t) \equiv 0$, the state trajectory of system (28) exhibits the chaotic behavior. Supposing that $x_1(t) \in [c - d, c + d]$ and $d = 10$, the following fuzzy systems (29) can exactly represent the nonlinear systems (28),

Rule 1: If $x_1(t)$ is M_1, then

$$\dot{x}(t) = A_1 x(t) + B u^{\mathrm{F}}(t),$$

Rule 2: If $x_1(t)$ is M_2, then

$$\dot{x}(t) = A_2 x(t) + B u^{\mathrm{F}}(t). \tag{29}$$

Here, $x(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}}$, and some related matrices and the membership functions are

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$h_1(t) = M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{c - x_1(t)}{d}\right),$$

$$h_2(t) = M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{c - x_1(t)}{d}\right).$$

Considering the three cases about initial state of system (29): a known initial state $x(0) = (1, -1, -1)^{\mathrm{T}}$, the initial state domains $\Omega_1 = \mathrm{co}\{(1, -1, -1)^{\mathrm{T}}, (1, 1, -1)^{\mathrm{T}}, (-1, 1, 1)^{\mathrm{T}}\}$, and $\Omega_2 = \{x : xRx \leqslant 1, x \in \mathbb{R}^n, R = 0.33 I_3\}$, and by the three design methods ((20), (23) and (27)), we can obtain three kinds of reliable guaranteed costs for fuzzy system (29) (see Table 1). By the optimization technique (20), we obtain the reliable fuzzy controller (6) with gain matrices (30) for fuzzy system (29).

$$F_1 = (20.7421 \quad 4.1770 \quad -35.2322),$$
$$F_2 = (20.6449 \quad 4.1575 \quad -34.9472). \quad (30)$$

Table 1. Reliable guaranteed costs with respect to different kinds of initial state domain

Initial state	$x(0)$	Ω_1	Ω_2
Guaranteed cost	$J^*(x(0)) = 297.3287$	$J^*(\Omega_1) = 526.4123$	$J^*(\Omega_2) = 626.8368$

When an actuator failure model (2) is chosen as $u^F(t) = 0.9u(t) + 0.1u(t)\sin(u(t))$, the state trajectory of the Rossler system (28) with the above reliable fuzzy controller is shown in Fig. 1. It is easy to see that the state trajectory is guided quickly to origin.

For the initial state domain Ω_1 and four kinds of actuator failures, we can obtain domain reliable guaranteed costs and gain matrix norms of the fuzzy controllers of the systems (29) (see Table 2). It is easy to see that the DRGCs and the controller's sizes all gradually increase with the increment of the degree of the actuator failures (2).

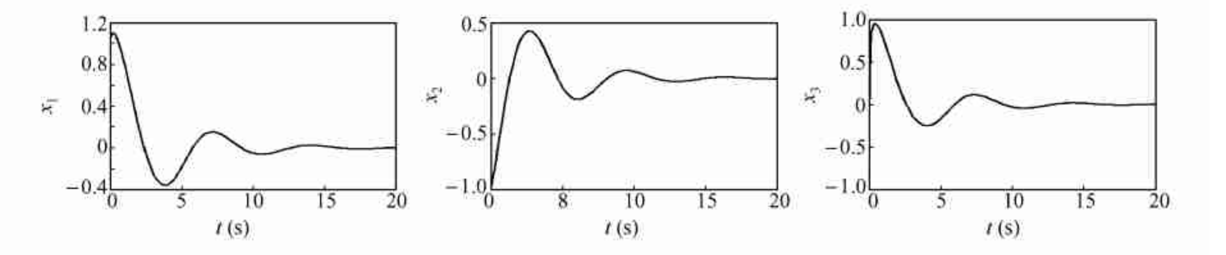

Fig. 1. State trajectories of the closed-loop Rossler system.

Table 2. Guaranteed costs and control gain norms with respect to different kinds of actuator failures

Failure parameters	$(\bar{\alpha}, \hat{\alpha}) =$	(1, 0.1)	(1, 0.2)	(0.9, 0.1)	(0.9, 0.2)
Guaranteed cost	$J^*(\Omega_1) =$	418.5052	527.1252	526.4123	683.9373
Gain norm	$(\|F_1\|, \|F_2\|) =$	(39.8498, 39.8401)	(44.8890, 44.7965)	(44.7351, 44.7623)	(51.0822, 51.1025)

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